Research Statement

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My research interests lie in the theoretical and algorithmic foundations of data science and applied harmonic analysis. The presented contributions are in three domains: 1) New structured approaches for physical layer security; 2) A statistical framework to explain the stability limits of continuous measurement operators; 3) The theory and algorithmic of continuous inverse problems and their applications to imaging, radar, and communication systems.

1 Physical Layer Security with Latent Structures

While communication privacy is often ensured at higher network layers and can be achieved via cryptographic means, there are new methods in *physical layer security* [1], which can leverage the structural properties of a communication channel to generate privacy. Traditional physical layer privacy strategies involve using artificial noise [2], injected either on the message or in the nullspace of the channel to harden eavesdropping. However, noise injection schemes rely on discriminative assumptions between the receiver and the eavesdropper channels, which is often impractical.

Instead of injecting noise, privacy can be achieved by relying on a secret prior on the transmitted message. For instance, the hardness of blind deconvolution, a bilinear inverse problem seeking to recover the two operands from the observation of their convolution, has been extensively leveraged for wireless privacy. More generally, if the communication channel is linear, the encoder (Alice) can establish a provably private communication with a legitimate receiver (Bob) by encoding the message in an instance of an inverse problem that is *identifiable* to Bob, but *unidentifiable* to an eavesdropper (Eve). We made the following contribution to that area.

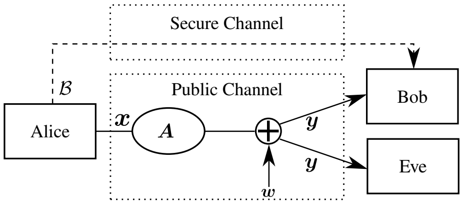

Figure 1: Communication with secure channel.

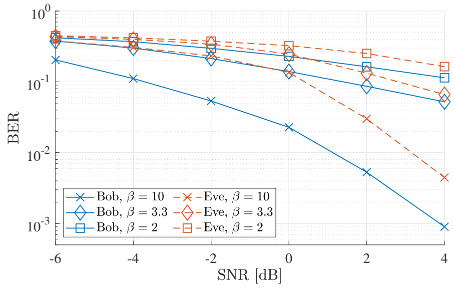

Figure 2: Bit error rate as a function of the SNR for different transmission parameters β.

Privacy via a secret block structure : Taking advantage of the fact that block-sparse signals can be perfectly reconstructed from few linear measurements in conditions where unstructured compressed sensing would probably fail [3], we propose a novel private communication framework where Alice achieves secrecy by transmitting instances of an unidentifiable compressed sensing problem over a public channel. Bob can attempt to overcome this ill-posedness by leveraging the secret knowledge of a block structure $\mathcal{B}$ that was used to encode Alice's message. Figure 1 sketches of the model. Contrary to comparable communication schemes, our framework has low usage of the secure information channel and only imposes an incoherence assumption on the channel matrix, which is *not* left for Alice to design and is typically imposed by the environment. Furthermore, our scheme can be implemented on top of existing overloaded CDMA and MIMO wireless channels with minimal hardware changes. Theoretical guarantees on the privacy of the communication are provided, and an upper bound on the number of messages that Alice can privately transmit *without refreshing the block structure* before it is statistically recoverable by Eve is established [A1]. The bit error rate after 500 key reuses is evaluated in Figure 2 for both Bob and Eve. This shows the privacy enhancements induced by the communication framework.

2 Extremization in the Bandlimit: From Harmonic Analysis to Statistical Resolution Limits

The Beurling–Selberg extremal approximation problem is a classical problem in functional analysis and has found applications in numerous areas of mathematics, including probability, dynamical systems, combinatorics, sphere packing, and sampling theory [4]. It consists of finding a majorant and a minorant to the signum function whose Fourier transforms are compactly supported within the interval $[-1, 1]$, and that achieve the best approximation to the signum function in the L_1-sense. Among other applications, the solutions to the Beurling–Selberg problem have been used to provide sharp bounds on the extremal singular values of non-harmonic Fourier matrices. Controlling the conditioning of such matrices is critical for engineering and imaging science applications, as system actions are often modeled by shift-invariant point-spread function (PSF).

Generalized Beurling problems and the stability of measurements operators : Under a shift-invariance assumption, the stability of this inversion is intrinsically ruled by the *spectral conditioning* of the measurement operator $\boldsymbol{A} = \mathrm{diag}(\boldsymbol{g})\boldsymbol{\Phi}$, where $\boldsymbol{g}$ encompasses the trigonometric moments of the PSF and $\boldsymbol{\Phi}$ is a *non-harmonic* Fourier matrix encoding source locations. In [A2], we unveil for the first time a relationship between *extremal singular values* of $\boldsymbol{A}$ and the Beurling–Selberg approximants of the autocorrelation of the PSF. Additionally, we propose novel harmonic analysis problems, which we call *second-order Beurling–Selberg approximation problems*, where approximation residuals to functions of bounded variations are constrained to faster decay rates in the asymptotic (see Figure 3). A relationship with the extremal eigenvalues of the Fisher information matrix of the signal parameters is established. This enables a *simple and universal characterization of the resolution limit* of a given measurement device.

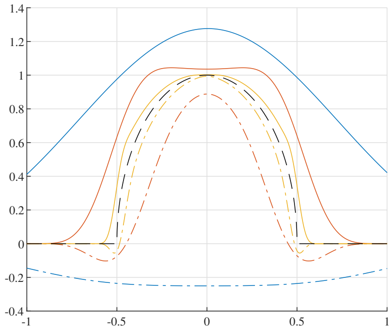

Figure 3: Second-order Beurling approximation to the semi-circular function, for different bandwidth.

Resolution limits of the Beurling-LASSO : Besides the above statistical bound, we also study the performance of the Beurling-LASSO estimator (*a.k.a.* atomic denoiser) [5], [A3]—a sparsity promoting convex estimator over continuous domains—to perform this inversion. In the absence of noise, we prove it is *necessary* the minimal distance between the point sources is greater than the inverse of the cutoff frequency of the PSF for the Beurling-LASSO to be tight [A4]. This work was shortlisted as one of the six finalists of the *IEEE Jack Keil Wolf Award*. The sufficiency of this limit remains a challenging open problem with profound theoretical implications in data science, of which the tightness of the statistical learning of mixtures from convex estimators, or the identifiability of blind super-resolution in imaging science. Furthermore, when the observations are noisy, it is critical to ensure the *support stability* of an estimator [6], where the deconvolution does not lead to spurious or missing estimates of the point sources. In [A5], we established the *stable resolution limit* of the Beurling-LASSO. For the first time, the link with the structural properties of the PSF was derived. Figure 4 compares our theoretical limit with the empirical stability of the Beurling-LASSO when the PSF is an ideal low-pass filter.

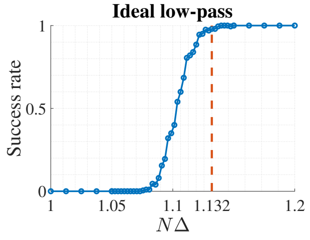

Figure 4: Support stability of the Beurling-LASSO

3 Harnessing Sparsity over the Continuum: Theory, Algorithms, and Applications

A versatile formulation to this class of problems discussed in Section 2 is to reconstruct a *continuous* Radon measure that both sufficiently explains the observation and exhibits a parsimonious structure over a predetermined continuous dictionary. Considering such inverse problems over the continuum raises new challenges: On the theoretical side, the inevitable spatial correlation of the dictionary elements makes the problem inexplicable by the classical theory of finite-dimensional inverse problems. Thus, theoretical guarantees demand a whole new framework of analysis. On the computational side, it requires new algorithms handling the reconstruction *directly over continuous domains*. However, removing the grid also brings many theoretical and numerical improvements: The absence of gridding avoids the risk of basis mismatch and the appearance of spurious artifacts inherent to the discretization. Moreover, the theoretical analysis of gridless methods grants a better understanding of the performance and stability of finite-dimensional models such as LASSO and leads to the development of new numerical methods that take advantage of the problem structure. We made the following theoretical, algorithmic, and applicative contributions to the area.

Computational complexity of the continuum and accelerations : If convex methods for solving inverse problems over the space of measures have appealing benefits over discretized ones, their computational complexity scales often polynomially with the number of observations n, which remains prohibitive for real-time implementations. When the spikes are positive, a provable compressed semidefinite program is proposed in [A6] of dimension $\max(r+1, \mathcal{O}(\sqrt{n}))$ to recover r sources, yielding *order of magnitude* improvement in the computational complexity when compared to the Beurling-LASSO. In [A7], we propose a highly scalable first-order method to deconvolve sparse measures based on *scaled gradient descent*. Despite the presence of local minima in the landscape, we guarantee the convergence towards the desired minimum at a linear rate that is independent of the conditioning of the problem.

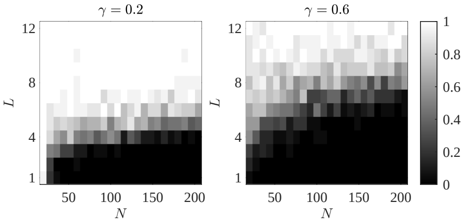

Figure 5: Success rate of the blind calibration algorithm for two different values of the sparsity level γ.

Self-calibration of time-varying system : The classical formulation of the sparse deconvolution problem assumes perfect knowledge or stringent assumptions on the point-spread function (PSF). When the PSF is subject to drift between batches of experiments (*e.g.* in microscopy) or is non-stationary (*e.g.* in radar imaging), the deconvolution presupposes thoughtfully calibrating the measurement device before acquiring the observations, which is often impractical. In the event of multichannel observations, the experimenter may attempt to recover both the PSF and the measures at once, yielding a bilinear inverse problem known as *multichannel blind calibration*. For this scenario, we proposed a novel adaptation of the Beurling-LASSO estimator [A8], and characterized its phase transition (*c.f.* Figure 5). Unlike most of the existing algorithms, which require either a latent low-dimensional structure on the PSF [7] or the uncorrelatedness of the input measures [8], our method makes minimal assumptions on those operands.

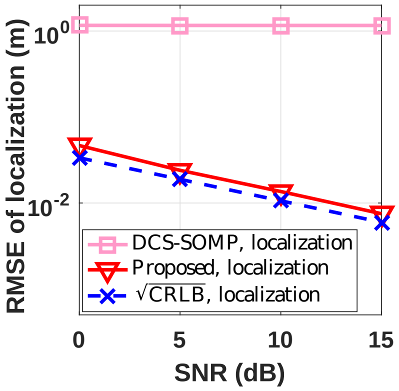

Figure 6: Localization performance of mm-Wave MIMO-OFDM probing.

Joint sensing and communication : An emerging challenge in wireless communication is to tame the sub-TeraHertz bandwidth. Because of its millimetric wavelengths, sub-teraHertz communication induces few propagation paths, making it well-mattered for localization and mapping of indoor environments. In [A9], we propose a novel off-the-grid approach to estimate the location and orientation of targets from millimeter-wave MIMO systems probing signals. While prior algorithms discretize the search space, our method harnesses jointly and continuously time-of-flight and angles of departure/arrival information yielding Cramér-Rao lower bound (CRLB) achieving performance (*c.f.* Fig 6), with considerable improvement with respect to the state-of-the-art methods.

4 Research Vision and Future Directions

The recent theoretical and algorithmic improvements on continuous inverse problems significantly impacted the sensing, imaging, and communication fields. Nevertheless, the assumptions considered in theory often remain stringent and unfit for real-world systems. Extending the theory of inverse problems to more versatile priors and measurement operators is of interest to partitioners in those applicative fields. We pinpoint in the sequel a list of investigation topics:

Towards provable and integrated physical layer security mecanisms : While the theoretical benefits brought by physical layer security protocols are promising, there is a need to integrate such approaches in realizable wireless and distributed systems [1], [9]. Building on our recent advances [A1], we shall explore whether more sophisticated structures than block sparsity can be harnessed and assess their privacy benefits. Additionally, we shall go beyond point-to-point communications and investigate how latent structures can secure networks of distributed agents.

Continuous reconstruction beyond shift-invariance : The bulk of the literature on continuous inverse problems focuses on deconvolving signals distorted by linear shift-invariant systems. However, many measurement operators occurring in imaging and microscopy (*e.g.* Laplace measurements, wavelet transforms, Gabor and Wilson frames, width-adaptive Gaussian frames) do not satisfy this assumption. The harmonic analysis underpinning presented in Section 2 could be extended to time-frequency operators, yielding a characterization of the resolution limits of a much broader class of measurement operators.

On the algorithmic aspects, most state-of-the-art methods still rely on problem discretization and suffer from inherent mismatch and sub-optimality issues. Recent trends in machine learning investigate over-parameterized gradient flow methods [10] as a sparsity-promoting regularizer for training wide neural networks. Linear convergence as those approaches towards a local minimum has been proven. We shall leverage this prior analysis to establish the convergence of first-order methods in continuous inverse problems.

Machine learning-aided non-linear inversion : The usual framework of continuous inverse problems assumes that the system has a PSF acting linearly on the input signal. Yet, the response of some practical systems, such as neural recording, exhibits non-negligible *non-linearities,* limiting PSF-based system modeling. Recent learning-based approaches train neural networks to approximate directly the inverse operator [11], opening new perspectives. However, those actual architectures lose most of the essence and geometry of the shift-invariant structure, demanding unreasonably large amounts of training samples to compensate. We shall investigate how neural network architecture inspired by classical signal processing algorithms (*e.g.* ESPRIT, Beurling-LASSO) can help better capture the problem structure and diminish the sample complexity of the training.

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